UNIT-I

Introduction and Fundamentals
- Motivation and Perspective
- Applications
- Components of Image Processing System
- Element of Visual Perception
- A Simple Image Model
- Sampling and Quantization

Image Enhancement in Frequency Domain
- Fourier Transform and the Frequency Domain
- Basis of Filtering in Frequency Domain
- Filters – Low-pass
  - High-pass
- Correspondence Between Filtering in Spatial and Frequency Domain
- Smoothing Frequency Domain Filters – Gaussian Low pass Filters
- Sharpening Frequency Domain Filters – Gaussian High pass Filters
- Homomorphic Filtering.
WHAT IS DIGITAL IMAGE PROCESSING?

An image may be defined as a two-dimensional function, \( f(x, y) \), where \( x \) and \( y \) are spatial (plane) coordinates, and the amplitude of at any pair of coordinates \( (x, y) \) is called the intensity or gray level of the image at that point. When \( x, y \), and the amplitude values are all finite, discrete quantities, we call the image a digital image. The field of digital image processing refers to processing digital images by means of a digital computer.

APPLICATIONS

The areas of application of digital image processing are so varied that some form of organization is desirable in attempting to capture the breadth of this field. One of the simplest ways to develop a basic understanding of the extent of image processing applications is to categorize images according to their source (e.g., visual, X-ray, and so on). Images based on radiation from the EM spectrum are the most familiar, especially images in the X-ray and visual bands of the spectrum. Each massless particle contains a certain amount (or bundle) of energy. Each bundle of energy is called a photon. If spectral bands are grouped according to energy per photon, we obtain the spectrum.
The electromagnetic spectrum arranged according to energy per photon.

1. Gamma-Ray Imaging

Major uses of imaging based on gamma rays include nuclear medicine. In nuclear medicine, the approach is to inject a patient with a radioactive isotope that emits gamma rays as it decays.

Bone scanning
2. X-ray Imaging

X-rays are among the oldest sources of EM radiation used for imaging. The best known use of X-rays is medical diagnostics.

3. Imaging in the Ultraviolet Band

Applications of ultraviolet “light” are varied. They include lithography, industrial inspection, microscopy, lasers, biological imaging.
4. Imaging in the Visible and Infrared Bands

Considering that the visual band of the electromagnetic spectrum is the most familiar in all our activities,

The examples range from pharmaceuticals and micro inspection to materials Characterization.

Material Inspection

5. Imaging in the Microwave Band

The dominant application of imaging in the microwave band is radar. The unique feature of imaging radar is its ability to collect data over virtually any region at any time, regardless of weather or ambient lighting conditions.

Radar image of mountains
FUNDAMENTAL STEPS IN DIGITAL IMAGE PROCESSING

It is helpful to divide the material covered in the following chapters into the two broad categories:

1. Methods whose input and output are images
2. Methods whose inputs may be images, but whose outputs are attributes extracted from those images.

Outputs of these processes generally are images

Outputs of these processes generally are image attributes
• *Image acquisition* is the first process. The discussion gave some hints regarding the origin of digital images.

• *Image enhancement* is among the simplest and most appealing areas of digital image processing. Basically, the idea behind enhancement techniques is to bring out detail that is obscured, or simply to highlight certain features of interest in an image.

• *Image restoration* is an area that also deals with improving the appearance of an image.

• *Color image processing* is an area that has been gaining in importance because of the significant increase in the use of digital images over the Internet.

• *Wavelets* are the foundation for representing images in various degrees of Resolution.

• *Compression*, as the name implies, deals with techniques for reducing the storage required to save an image.

• *Morphological processing* deals with tools for extracting image components that are useful in the representation and description of shape.

• *Segmentation* procedures partition an image into its constituent parts or objects.

• *Representation and description* almost always follow the output of a segmentation stage, which usually is raw pixel data, constituting either the boundary of a region.

➤ **COMPONENTS OF AN IMAGE PROCESSING SYSTEM**

The basic components comprising a typical *general-purpose* system used for digital image processing.
Problem domain

Components of a general-purpose image processing system.

Image sensing two elements are required to acquire digital images. The first is a physical device that is sensitive to the energy radiated by the object we wish to image. The second, called a digitizer, is a device for converting. The output of the physical sensing device into digital form.

Specialized image processing hardware usually consists of the digitizer just mentioned, plus hardware that performs other primitive operations, such as an arithmetic logic unit (ALU), which performs arithmetic and logical operations in parallel on entire images.

Computer in an image processing system is a general-purpose computer and can range from a PC to a supercomputer.

Software for image processing consists of specialized modules that perform specific tasks.
Mass storage capability is a must in image processing applications. An image of size 1024*1024 pixels, in which the intensity of each pixel is an 8-bit quantity.

Hardcopy devices for recording images include laser printers, film cameras, heat-sensitive devices, inkjet units, and digital units, such as optical and CD-ROM disks.

- **ELEMENTS OF VISUAL PERCEPTION**

Although the digital image processing field is built on a foundation of mathematical and probabilistic formulations and play a central role in the choice of one technique versus another.

1. **Structure of the Human Eye**

Figure shows a simplified horizontal cross section of the human eye. The eye is nearly a sphere, with an average diameter of approximately 20 mm.

Three membranes enclose the eye: the cornea and sclera outer cover; the choroid; and the retina.
There are two classes of receptors: cones and rods. The cones in each eye number between 6 and 7 million. They are located primarily in the central portion of the retina, called the fovea, and are highly sensitive to color. Humans can resolve fine details with these cones largely because each one is connected to its own nerve end.

2. Image Formation in the Eye

The principal difference between the lens of the eye and an ordinary optical lens is that the former is flexible.

The distance between the center of the lens and the retina (called the focal length) varies from approximately 17 mm to about 14 mm.

\[
f(x,y) = \text{reflectance}(x,y) \times \text{illumination}(x,y)
\]

Reflectance in \([0,1]\), illumination in \([0,\infty]\)
3. Brightness Adaptation and Discrimination

Because digital images are displayed as a discrete set of intensities, the eye’s ability to discriminate between different intensity levels is an important consideration in presenting Image-processing results.
Image Sensing and Acquisition

The types of images in which we are interested are generated by the combination of an “illumination” source and the reflection or absorption of energy from that source by the elements of the “scene” being imaged. We enclose illumination and scene in quotes to emphasize the fact that they are considerably more general than the familiar situation in which a visible light source illuminates a common everyday 3-D (three-dimensional) scene.

1. A Simple Image Formation Model

As introduced in Section 1.1, we shall denote images by two-dimensional functions of the form f(x, y). The value or amplitude of f at spatial coordinates (x, y) is a positive scalar quantity whose physical meaning is determined by the source of the image.

\[ 0 < f(x, y) < q. \]

The function f(x, y) may be characterized by two components: (1) the amount of source illumination incident on the scene being viewed, and (2) the amount of illumination reflected by the objects in the scene. Appropriately, these are called the illumination and reflectance components and are denoted by i(x, y) and r(x, y), respectively. The two functions combine as a product to form f(x, y):

\[ f(x, y) = i(x, y)r(x, y) \]

\[ \text{Where} \quad 0 < i(x, y) < q \]

\[ \text{And} \quad 0 < r(x, y) < 1. \]

Equation indicates that reflectance is bounded by 0 (total absorption) and 1 (total reflectance). The nature of i(x, y) is determined by the illumination source, and r(x, y) is determined by the characteristics of the imaged objects.

Image Sampling and Quantization

To generate digital images from sensed data, the output of most sensors is a continuous voltage waveform whose amplitude and spatial behavior are related to the physical phenomenon being sensed. To create a digital image, we need to convert the continuous sensed data into digital form. This involves two processes: sampling and quantization.

The basic idea behind sampling and quantization is a continuous image, f(x, y), that we want to convert to digital form. An image may be continuous with respect to the x- and y-coordinates, and also in amplitude. To convert it to digital form, we have to sample the function in both
coordinates and in amplitude. Digitizing the coordinate values is called \textit{sampling}. Digitizing the amplitude values is called \textit{quantization}.

(a) Continuous image. (b) A scan line from $A$ to $B$ in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

(b) is a plot of amplitude (gray level) values of the continuous image along the line segment $AB$ in (a). The random variations are due to image noise. To sample this function, we take equally spaced samples along line $AB$, as shown in (c). The location of each sample is given by a vertical tick mark in the bottom part of the figure.
Representing Digital Images

The result of sampling and quantization is a matrix of real numbers. We will use two principal ways in this book to represent digital images. Assume that an image \( f(x, y) \) is sampled so that the resulting digital image has \( M \) rows and \( N \) columns. The values of the coordinates \( (x, y) \) now become *discrete* quantities.

**Coordinate convention used to represent digital images.**
The notation introduced in the preceding paragraph allows us to write the complete M*N digital image in the following compact matrix form:

\[
f(x, y) = \begin{bmatrix}
f(0, 0) & f(0, 1) & \cdots & f(0, N - 1) \\
(f(1, 0)) & f(1, 1) & \cdots & f(1, N - 1) \\
\vdots & \vdots & \ddots & \vdots \\
f(M - 1, 0) & f(M - 1, 1) & \cdots & f(M - 1, N - 1)
\end{bmatrix}.
\]

The right side of this equation is by definition a digital image. Each element of this matrix array is called an image element, picture element, pixel, or pel. The terms image and pixel will be used throughout the rest of our discussions to denote a digital image and its elements.

Now, we have to simple write as…

\[
A = \begin{bmatrix}
a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\
a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\
\vdots & \vdots & \ddots & \vdots \\
a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1}
\end{bmatrix}.
\]

Clearly, \(a(i,j) = f(x=i, y=j) = f(i, j)\), so matrix \(f(x,y)\) and \(A\) is identical matrix.

Expressing sampling and quantization in more formal mathematical terms can be useful at times.

Let \(Z\) and \(R\) denote the set of real integers and the set of real numbers, respectively. The sampling process may be viewed as partitioning the \(x,y\) plane into a grid, with the coordinates of the center of each grid.

This digitization process requires decisions about values for \(M\), \(N\), and for the number, \(L\), of discrete gray levels allowed for each pixel. There are no requirements on \(M\) and \(N\), other than that they have to be positive integers.

the number of gray levels typically is an integer power of 2:

\[
L = 2^k.
\]
We assume that the discrete levels are equally spaced and that they are integers in the interval [0, L-1]. Sometimes the range of values spanned by the gray scale is called the *dynamic range* of an image.

When an appreciable number of pixels exhibit this property, the image will have high contrast. Conversely, an image with low dynamic range tends to have a dull, washed out gray look.

The number, \( b \), of bits required to store a digitized image is

\[
b = M \times N \times k.
\]

If this is square image then \( M=N \)

\[
b = N^2 k.
\]

K is use as “k-bit image”.

**Spatial and Gray-Level Resolution**

Sampling is the principal factor determining the *spatial resolution* of an image. Basically, spatial resolution is the smallest discernible detail in an image.

*Gray-level resolution* similarly refers to the smallest discernible change in gray level. When an actual measure of physical resolution relating pixels and the level of detail they resolve in the original scene are not necessary, it is not uncommon to refer to an L-level digital image of size M*N as having a spatial resolution of M*N pixels and a gray-level resolution of L levels. We will use this terminology from time to time in subsequent discussions.

**Typical effects of varying the number of samples in a digital image.**
Some Basic Relationships Between Pixels

Neighbors of a Pixel

A pixel p at coordinates (x, y) has four horizontal and vertical neighbors whose coordinates are given by

\[(x + 1, y), (x - 1, y), (x, y + 1), (x, y - 1)\]

This set of pixels, called the 4-neighbors of p, is denoted by \(N_4(p)\). Each pixel is a unit distance from (x, y), and some of the neighbors of p lie outside the digital image if (x, y) is on the border of the image.

The four diagonal neighbors of p have coordinates

\[(x + 1, y + 1), (x + 1, y - 1), (x - 1, y + 1), (x - 1, y - 1)\]

and are denoted by \(N_D(p)\). These points, together with the 4-neighbors, are called the 8-neighbors of p, denoted by \(N_8(p)\).

Adjacency, Connectivity, Regions, and Boundaries

Connectivity between pixels is a fundamental concept that simplifies the definition of numerous digital image concepts, such as regions and boundaries. To establish if two pixels are connected, it must be determined if they are neighbors.

We consider 3 type of adjacencies;

(a) 4-adjacency. Two pixels p and q with values from \(V\) are 4-adjacent if q is in the set \(N_4(p)\).

(b) 8-adjacency. Two pixels p and q with values from \(V\) are 8-adjacent if q is in the set \(N_8(p)\).

(c) m-adjacency (mixed adjacency). Two pixels p and q with values from \(V\) are m-adjacent.

Two pixels p and q are said to be connected in \(S\) if there exists a path between them consisting entirely of pixels in \(S\). For any pixel p in \(S\), the set of pixels that are connected to it in \(S\) is called a connected component of \(S\). If it only has one connected component, then set \(S\) is called a connected set.
(a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) m-adjacency.

Distance Measures

For pixels p, q, and z, with coordinates (x, y), (s, t), and (v, w), respectively, D is a distance function or metric if

(a) $D(p, q) \geq 0 \quad (D(p, q) = 0 \quad \text{iff} \quad p = q)$,
(b) $D(p, q) = D(q, p)$, and
(c) $D(p, z) \leq D(p, q) + D(q, z)$.

The Euclidean distance between p and q is defined as

$$D_e(p, q) = \left[ (x - s)^2 + (y - t)^2 \right]^{\frac{1}{2}}.$$

The $D_4$ distance (also called city-block distance) between p and q is defined as

$$D_4(p, q) = |x - s| + |y - t|.$$

The pixels with $D_4=1$ are the 4-neighbors of (x, y).
The **D8 distance** (also called *chessboard distance*) between p and q is defined as

\[ D_8(p, q) = \max(|x - s|, |y - t|). \]

\[ \begin{array}{cccccc}
2 & 2 & 2 & 2 & 2 \\
2 & 1 & 1 & 1 & 2 \\
2 & 1 & 0 & 1 & 2 \\
2 & 1 & 1 & 1 & 2 \\
2 & 2 & 2 & 2 & 2 \\
\end{array} \]

The pixels with \(D_8=1\) are the 8-neighbors of \((x, y)\).

Note that the \(D_4\) and \(D_8\) distances between p and q are independent of any paths that might exist between the points because these distances involve only the coordinates of the points.


- **IMAGE ENHANCEMENT IN FREQUENCY DOMAIN**

**Preliminary concepts**

- Complex numbers

A complex number $C$ is defined by

$$C = R + jI$$

The conjugate of a complex number $C$, denoted by $C_\_\_\_$ is defined by

$$C_\_\_\_\_\_\_ = R - jI$$

May be viewed geometrically as points on a plane (complex plane)

- Abscissa is the real axis (value of $R$)
- Ordinate is the imaginary axis (value of $I$)

$C$ can be represented as point $(R, I)$ in the rectangular coordinate system of complex plane

**Polar representation of complex numbers** is given by

$$C = |C| (\cos \theta + j \sin \theta)$$

- $|C| = \sqrt{R^2 + I^2}$ is the magnitude of the vector from the origin to point $(R, I)$
- $\theta$ is the angle between the vector and the real axis

$$\tan \theta = I/R \Rightarrow \theta = \tan^{-1}(I/R)$$

Using Euler’s formula

$$e^{j\theta} = \cos \theta + j \sin \theta$$

Then, the complex number is given by

$$C = |C| e^{j\theta}$$

The complex function $F(u)$ can be expressed as the sum $F(u) = R(u) + jI(u)$ where $R(u)$ and $I(u)$ are real and imaginary component functions.
A function $f(t)$ of a continuous variable $t$ that is periodic with period $T$ is expressed as follows:

$$ f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n t}{T}} $$

where

$$ c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi n t}{T}} \, dt \quad \text{for} \ n = 0, \pm 1, \pm 2, \ldots $$

**ONE-DIMENSIONAL (1-D) FOURIER TRANSFORM**

Given single variable continuous function $f(t)$ of a continuous variable $t$

Fourier transform $F(u)$ is given by

$$ \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j2\pi \mu t} \, dt $$

* $\mu$ is also a continuous variable
* $t$ is integrated out, so, $\mathcal{F}\{f(t)\}$ is a function only of $\mu$
* So, we can write $\mathcal{F}\{f(t)\} = F(\mu)$ or

$$ F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi \mu t} \, dt $$

The inverse of the Fourier transform is given by

$$ f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi \mu t} \, d\mu $$

Fourier transform can be expressed using Euler’s formula as

$$ F(\mu) = \int_{-\infty}^{\infty} f(t) [\cos(2\pi \mu t) - j\sin(2\pi \mu t)] \, dt $$

Since $F(u)$ is Complex, therefore

$F(u)=R(u)+j\ I(u)$

Where $R$ is real and $I$ is imaginary.
\[ F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux + vy)} \, dx \, dy \]

and, similarly for the inverse transform,
\[ f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux + vy)} \, du \, dv. \]

\[ F(u) = |F(u)| e^{-j\phi(u)} \]

where
\[ |F(u)| = \left[ R^2(u) + I^2(u) \right]^{1/2} \]

is called the magnitude or spectrum of the Fourier transform,
\[ \phi(u) = \tan^{-1} \left( \frac{I(u)}{R(u)} \right) \]

is called the phase angle or phase spectrum of the transform.

Power spectrum is defined as square of fourier transform……
\[ P(u) = |F(u)|^2 = R^2(u) + I^2(u). \]

The term spectral density also is used to refer to the power spectrum.
TWO-DIMENSIONAL DFT AND ITS INVERSE

In 2D, DFT of an image \( f(x, y) \) of dimensions \( M \times N \) is given by

\[
F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M+vy/N)}
\]

The inverse Fourier transform is given by

\[
f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M+vy/N)}
\]

Fourier spectrum, phase angle, and power spectrum are defined by

\[
|F(u, v)| = \left[ R^2(u, v) + I^2(u, v) \right]^{1/2}
\]
\[
\phi(u, v) = \tan^{-1} \left( \frac{I(u, v)}{R(u, v)} \right)
\]
\[
P(u, v) = |F(u, v)|^2 = R^2(u, v) + I^2(u, v)
\]

At point \((0, 0)\) we have

\[
F(0, 0) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)
\]

which is the average of the entire image.

If \( f(x, y) \) is real, its Fourier transform is conjugate symmetric; that is

\[
F(u, v) = F^*(-u, -v)
\]

where “*” indicates the standard conjugate operation on a complex. From this, it follows that

\[
|F(u, v)| = |F(-u, -v)|,
\]

which says that the spectrum of the Fourier transform is symmetric. C
Basics of filtering in the frequency domain

Filtering in the frequency domain is straightforward. It consists of the following steps:

1. Multiply the input image by \((-1)^{x+y}\) to center the transform, as indicated in Eq. (4.2-21).
2. Compute \(F(u, v)\), the DFT of the image from (1).
3. Multiply \(F(u, v)\) by a filter function \(H(u, v)\).
4. Compute the inverse DFT of the result in (3).
5. Obtain the real part of the result in (4).
6. Multiply the result in (5) by \((-1)^{x+y}\).

The reason that \(H(u, v)\) is called a filter (the term filter transfer function also is used commonly) is because it suppresses certain frequencies in the transform while leaving others unchanged. The analogy from everyday life is a screen filter that passes certain objects and suppresses others, based strictly on their size.

In equation form, let \(f(x, y)\) represent the input image in Step 1 and \(F(u, v)\) its Fourier transform. Then the Fourier transform of the output image is given by

\[
G(u, v) = H(u, v)F(u, v). \tag{4.2-27}
\]

The multiplication of \(H\) and \(F\) involves two-dimensional functions and is defined on an element-by-element basis. That is, the first element of \(H\) multiplies the first element of \(F\), the second element of \(H\) multiplies the second element of \(F\), and so on. In general, the components of \(F\) are complex quantities, but the filters with which we deal in this book typically are real. In this case, each component of \(H\) multiplies both the real and imaginary parts of the corresponding component in \(F\). Such filters are called zero-phase-shift filters. As their name

The filtered image is obtained simply by taking the inverse Fourier transform of \(G(u, v)\):

\[
\text{Filtered Image} = \mathcal{F}^{-1}[G(u, v)]. \tag{4.2-28}
\]
Some basic filters and their properties

At this point we have established the foundation for filtering in the frequency domain. The next logical step is to look at some specific filters and see how they affect images.

The average value of the image is given by $F(0, 0)$. If we set this term to zero in the frequency domain and take the inverse transform, then the average value of the resulting image will be zero.

**filter function:**

$$H(u, v) = \begin{cases} 
0 & \text{if } (u, v) = (M/2, N/2) \\ 
1 & \text{otherwise.} 
\end{cases}$$

All this filter would do is set $F(0, 0)$ to zero and leave all other frequency components of the Fourier transform untouched, as desired. The processed image (with zero average value) can then be obtained by taking the inverse Fourier transform of $H(u, v)F(u, v)$, as indicated in Eq. (4.2-28). As stated earlier, both the real and imaginary parts of $F(u, v)$ are multiplied by the filter function $H(u, v)$.

The filter just discussed is called a **notch filter** because it is a constant function with a hole (notch) at the origin.
Low frequencies in the Fourier transform are responsible for the general gray-level appearance of an image over smooth areas, while high frequencies are responsible for detail, such as edges and noise. These ideas are discussed in more detail in the sections that follow, but it will be instructive to complement our illustration of the notch filter with an example of filters in these other two categories. A filter that attenuates high frequencies while “passing” low frequencies is called a lowpass filter. A filter that has the opposite characteristic is appropriately called a highpass filter.
Now we will see the difference between low pass filtering and high pass filtering regarding the original image and its fourier transform (A), (B).

(B) FOURIER TRANSFORM OF DAMAGED I.C (ORIGINAL IMAGE)
(C) 2-D LOW PASS FILTER FUNCTION WITH RESULT OF LOWPASS FILTERING
(D) 2-D HIGH PASS FILTER FUNCTION WITH RESULT OF HIGH PASS FILTERING
CORRESPONDENCE BETWEEN FILTERING IN THE SPATIAL AND FREQUENCY DOMAINS

The most fundamental relationship between the spatial and frequency domains is established by a well-known result called the CONVOLUTION THEOREM.

Formally, the discrete convolution of two functions \( f(x, y) \) and \( h(x, y) \) of size \( M \times N \) is denoted by \( f(x, y) \ast h(x, y) \) and is defined by the expression

\[
f(x, y) \ast h(x, y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n)h(x - m, y - n).
\]

**Properties of convolution implementation:**

1. Flipping one function about the origin.
2. Shifting that function w.r.to other by changing the values of \((x, y)\).
3. Computing a sum of products over all values of \(m\) and \(n\).

Letting \( F(u, v) \) and \( H(u, v) \) denote the Fourier transforms of \( f(x, y) \) and \( h(x, y) \), respectively, one-half of the convolution theorem simply states that \( f(x, y) \ast h(x, y) \) and \( F(u, v)H(u, v) \) constitute a Fourier transform pair. This result is formally stated as

\[
f(x, y) \ast h(x, y) \iff F(u, v)H(u, v) .
\]

The double arrow is used to indicate that the expression on the left (spatial convolution) can be obtained by taking the **inverse** Fourier transform of the expression on the right can be obtained by taking the **forward** Fourier transform of the expression on the left. An analogous result is that convolution in the frequency domain reduces to multiplication in the spatial domain, and vice versa; that is,

\[
f(x, y)h(x, y) \iff F(u, v) \ast H(u, v).
\]

these two results comprise the convolution theorem.

Now we discuss the case of a unit impulse located at origin. Which is denoted as 

\[
\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} s(x, y)\delta(x, y) = s(0, 0).
\]
Now we can compute the Fourier transform of a unit impulse at the origin,

\[ F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \delta(x, y) e^{-j2\pi(ux/M + vy/N)} \]

\[ = \frac{1}{MN} \]

We suppose that \( f(x, y) = d(x, y) \) and carry out the convolution theorem……

\[ f(x, y) \ast h(x, y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \delta(m, n)h(x - m, y - n) \]

\[ = \frac{1}{MN} h(x, y) \]

Now we combining the previous two results with eq(4.2-31) and we obtain………..

\[ f(x, y) \ast h(x, y) \Leftrightarrow F(u, v)H(u, v) \]

\[ \delta(x, y) \ast h(x, y) \Leftrightarrow \mathcal{F}[\delta(x, y)]H(u, v) \]

\[ h(x, y) \Leftrightarrow H(u, v). \]

**SMOOTHING FREQUENCY-Domain FILTERS:-**

Edges and sharp transition (noise) in grey levels of an image contribute significantly to the high-frequency content of its Fourier transform. Hence smoothing (blurring) is achieved in the frequency domain by attenuating a specified range of high frequency components in the transform of a given image.

Our basic model for filtering in frequency domain is…

\[ G(u, v) = H(u, v)F(u, v) \]

where \( F(u, v) \) is the Fourier transform of the image to be smoothed. The objective is to select a filter transfer function \( H(u, v) \) that yields \( G(u, v) \) by attenuating the high-frequency components of \( F(u, v) \).
There are 3 types of low pass filters.

1. Ideal low pass filter
2. Butterworth low pass filter
3. Gaussian low pass filter

In syllabus, we will read only Gaussian low pass filters.

**GAUSSIAN LOWPASS FILTERS:**

Gaussian lowpass filters (GLPFs) of one dimension were introduced in Section 4.2.4 as an aid in exploring some important relationships between the spatial and frequency domains. The form of these filters in two dimensions is given by

\[
H(u, v) = e^{-D^2(u, v)/2\sigma^2}
\]

(4.3-7)

where, as in Eq. (4.3-3), \(D(u, v)\) is the distance from the origin of the Fourier transform, which we assume has been shifted to the center of the frequency rec-

(a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image.
\( \sigma \) is a measure of the spread of the Gaussian curve. By letting \( \sigma = D_0 \), we can express the filter in a more familiar form in terms of the notation in this section:

\[
H(u, v) = e^{-D^2(u,v)/2D_0^2}
\]  

where \( D_0 \) is the cutoff frequency. When \( D(u, v) = D_0 \), the filter is down to 0.607 of its maximum value.

(a) Original image.
(b)-(d) GLPF having cut off frequency 5, 15 and 30.
SHARPENING FREQUENCY DOMAIN FILTERS:

Image sharpening can be achieved in the frequency domain by a high-pass filtering process, which attenuates the low-frequency components without disturbing high-frequency information in the Fourier transform.

Because the intended function of the filters in this section is to perform precisely the reverse operation of the ideal lowpass filters discussed in the previous section, the transfer function of the highpass filters discussed in this section can be obtained using the relation

\[ H_{hp}(u, v) = 1 - H_{lp}(u, v) \]  \hspace{1cm} (4.4-1)

where \( H_{lp}(u, v) \) is the transfer function of the corresponding lowpass filter.

Gaussian Highpass Filters

The transfer function of the Gaussian highpass filter (GHPF) with cutoff frequency locus at a distance \( D_0 \) from the origin is given by

\[ H(u, v) = 1 - e^{-D^2(u,v)/2D_0^2} \]  \hspace{1cm} (4.4-4)
The Laplacian in the Frequency Domain

It can be shown that

$$\mathcal{F}\left[ \frac{d^n f(x)}{dx^n} \right] = (ju)^n F(u). \quad (4.4-5)$$

From this simple expression, it follows that

$$\mathcal{F}\left[ \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2} \right] = (ju)^2 F(u, v) + (jv)^2 F(u, v)$$

$$= -(u^2 + v^2)F(u, v). \quad (4.4-6)$$

The expression inside the brackets on the left side of Eq. (4.4-6) is recognized as the Laplacian of $f(x, y)$, defined in Eq. (3.7-1). Thus, we have the important result

$$\mathcal{F}[\nabla^2 f(x, y)] = -(u^2 + v^2)F(u, v), \quad (4.4-7)$$

which simply says that the Laplacian can be implemented in the frequency domain by using the filter

$$H(u, v) = -(u^2 + v^2). \quad (4.4-8)$$
$M \times N$, this operation shifts the center transform so that $(u, v) = (0, 0)$ is at point $(M/2, N/2)$ in the frequency rectangle. As before, the center of the filter function also needs to be shifted:

$$H(u, v) = -[(u - M/2)^2 + (v - N/2)^2].$$  \hspace{1cm} (4.4-9)

The Laplacian-filtered image in the spatial domain is obtained by computing the inverse Fourier transform of $H(u, v)F(u, v)$:

$$\nabla^2 f(x, y) = \mathcal{F}^{-1} \{-[(u - M/2)^2 + (v - N/2)^2]F(u, v)\}. \hspace{1cm} (4.4-10)$$

Conversely, computing the Laplacian in the spatial domain using Eq. (3.7-1) and computing the Fourier transform of the result is equivalent to multiplying $F(u, v)$ by $H(u, v)$. We express this dual relationship in the familiar Fourier-transform-pair notation

$$\nabla^2 f(x, y) \Leftrightarrow -[(u - M/2)^2 + (v - N/2)^2]F(u, v).$$  \hspace{1cm} (4.4-11)
Homomorphic Filtering

The illumination-reflectance model introduced in Section 2.3.4 can be used to develop a frequency domain procedure for improving the appearance of an image by simultaneous gray-level range compression and contrast enhancement as the product of illumination and reflectance components:

\[ f(x, y) = i(x, y)r(x, y). \]

of the product of two functions is not separable; in other words,

\[ \Imag{f(x, y)} \neq \Imag{i(x, y)}\Imag{r(x, y)}. \]

Suppose, however, that we define

\[ z(x, y) = \ln f(x, y) = \ln i(x, y) + \ln r(x, y). \]  \hspace{1cm} (4.5-2)

Then

\[ \Imag{z(x, y)} = \Imag{\ln f(x, y)} = \Imag{\ln i(x, y)} + \Imag{\ln r(x, y)} \]  \hspace{1cm} (4.5-3)

or

\[ Z(u, v) = F_i(u, v) + F_r(u, v) \]  \hspace{1cm} (4.5-4)

where \( F_i(u, v) \) and \( F_r(u, v) \) are the Fourier transforms of \( \ln i(x, y) \) and \( \ln r(x, y) \), respectively.

If we process \( Z(u, v) \) by means of a filter function \( H(u, v) \) then, from Eq. (4.2-27),

\[ S(u, v) = H(u, v)Z(u, v) = H(u, v)F_i(u, v) + H(u, v)F_r(u, v) \]  \hspace{1cm} (4.5-5)

where \( S(u, v) \) is the Fourier transform of the result. In the spatial domain,

\[ s(x, y) = \mathcal{F}^{-1}\{S(u, v)\} = \mathcal{F}^{-1}\{H(u, v)F_i(u, v)\} + \mathcal{F}^{-1}\{H(u, v)F_r(u, v)\}. \]  \hspace{1cm} (4.5-6)
By letting
\[ i'(x, y) = \mathcal{F}^{-1}\{H(u, v)F_i(u, v)\} \]  
(4.5-7)
and
\[ r'(x, y) = \mathcal{F}^{-1}\{H(u, v)F_r(u, v)\}, \]  
(4.5-8)
Eq. (4.5-6) can be expressed in the form
\[ s(x, y) = i'(x, y) + r'(x, y). \]  
(4.5-9)
Finally, as \( z(x, y) \) was formed by taking the logarithm of the original image \( f(x, y) \), the inverse (exponential) operation yields the desired enhanced image, denoted by \( g(x, y) \); that is,
\[ g(x, y) = e^{s(x, y)} = e^{i'(x, y)} \cdot e^{r'(x, y)} = i_0(x, y) r_0(x, y) \]  
(4.5-10)
where
\[ i_0(x, y) = e^{i'(x, y)} \]  
(4.5-11)

\[ \begin{align*}
 f(x, y) & \rightarrow \ln \rightarrow \text{DFT} \rightarrow H(u, v) \rightarrow \text{(DFT)}^{-1} \rightarrow \exp \rightarrow g(x, y)
\end{align*} \]

and
\[ r_0(x, y) = e^{r'(x, y)} \]  
(4.5-12)
are the illumination and reflectance components of the output image.
ORIGINAL IMAGE

PROCESSED BY H. FILTERING

- THE END-